MAT 2500 (Dr. Fuentes) Worksheet 5 - Sections 13.3 & 13.4

Section 13.3: Arc Length

Problem 1. Find the length of each of the following curves over the given range of t.

(a) $\mathbf{r}(t) = \langle 2t, t^2, \frac{1}{3}t^3 \rangle, \ 0 \le t \le 1$, (b) $\mathbf{r}(t) = \mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}, \ 0 \le t \le 1.$

Problem 2. Find the arc length function for the curve measured from the point P in the direction of increasing t and then reparametrize the curve with respect to arc length starting from P.

$$\mathbf{r}(t) = (5-t)\mathbf{i} + (4t-3)\mathbf{j} + 3t\mathbf{k}, \qquad P = (4,1,3).$$

Problem 3. Suppose you start at the point $(0,0,3)$ and move 5 units along the curve $\mathbf{r}(t) = 3\sin(t)\mathbf{i} + 4t\mathbf{j} + 3\cos(t)\mathbf{k}$ in the positive direction. At what point on the curve are you now?

HINT: The answer is not $(5,5,8)$ nor is it $\mathbf{r}(5) = \langle 3\sin(5), 32, 3\cos(5) \rangle$. You must locate the point at which the arc length equals 5 starting from the point $(0,0,3)$. Solve $\int_0^t |\mathbf{r}'(u)| \, \mathrm{du} = 5$ for t to find the point.

Section 13.4: Motion in Space: Velocity and Acceleration

Problem 4. Find the position vector of a function that has

acceleration vector $\mathbf{a}(t) = 2t\mathbf{i} + \sin(t)\mathbf{j} + \cos(2t)\mathbf{k}$,

initial velocity $\mathbf{v}(0) = \mathbf{i}$, and

initial position $\mathbf{r}(0) = \mathbf{j}$.

Problem 5. The position function of a moving particle is given by $\mathbf{r}(t) = \langle t^2, 5t, t^2 - 16t \rangle$. The unit of displacement is in meters (m) and the unit of time is in seconds (s). When is the speed a minimum?

HINT: The speed of the particle will give you a 1-variable function in terms of t. Use a Calculus 1 technique to minimize this function.