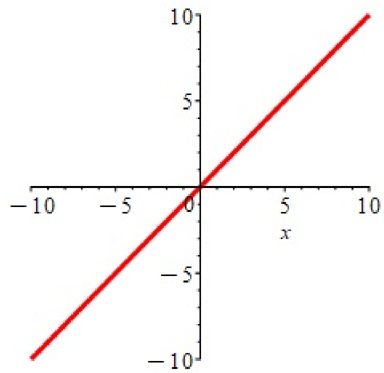
MAT 2500 (Dr. Fuentes) **Worksheet 1 - Section 12.1**

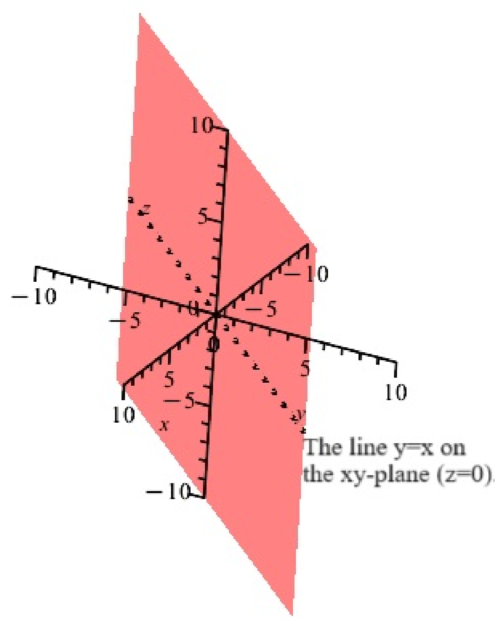
Section 12.1: 3-Dimensional Coordinate Systems

Problem 1. Consider the equation $y = x$.
(a) Describe the type of curve this equation represents in $\mathbb{R}^2$ (on the xy-plane). Draw a rough sketch of the curve.
(b) Describe the type of surface this equation represents in $\mathbb{R}^3$. Draw a rough sketch with axes in standard position.

(a) The equation represents the diagonal line $y = x$ on the xy-plane.

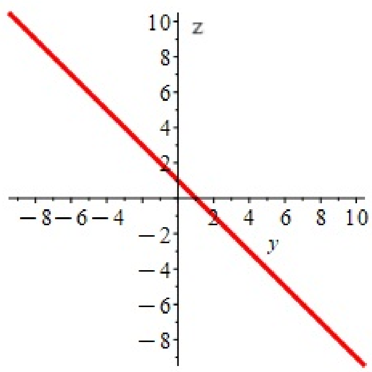


(b) The equation represents the vertical plane that passes through the line $y = x$ on the xy-plane.

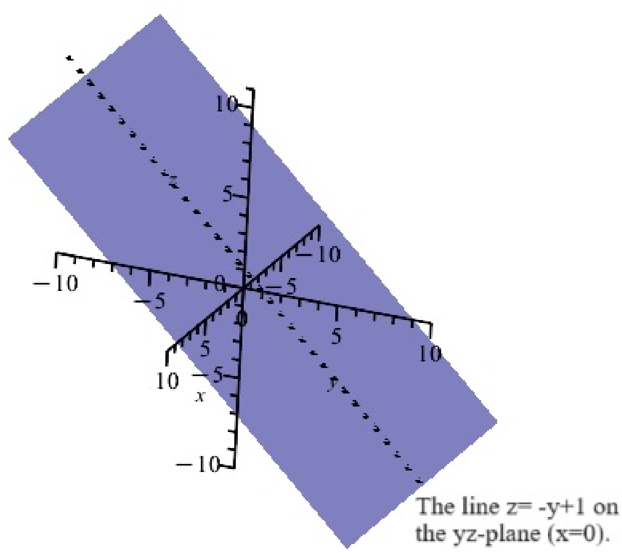


Problem 2. Consider the equation $y + z = 1$.
(a) Describe the type of curve this equation represents in $\mathbb{R}^2$ (on the yz-plane). Draw a rough sketch of the curve. **HINT:** Let the z-axis be vertical.
(b) Describe the type of surface this equation represents in $\mathbb{R}^3$. Draw a rough sketch with axes in standard position.

(a) The equation represents the diagonal line $z = -y + 1$ on the yz-plane.

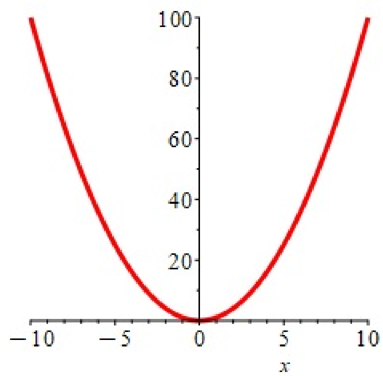


(b) The equation represents a diagonal plane that passes through the line $z = -y + 1$ on the yz-plane.



Problem 3. Consider the equation $z = x^2$.
(a) Describe the type of curve this equation represents in $\mathbb{R}^2$ (on the xz-plane). Draw a rough sketch of the curve. **HINT:** Let the z-axis be vertical.
(b) Draw a rough sketch of the surface with axes in standard position in $\mathbb{R}^3$. **This surface is called a parabolic cylinder in $\mathbb{R}^3$.**

(a) The equation represents the parabola $z = x^2$ on the xz-plane.



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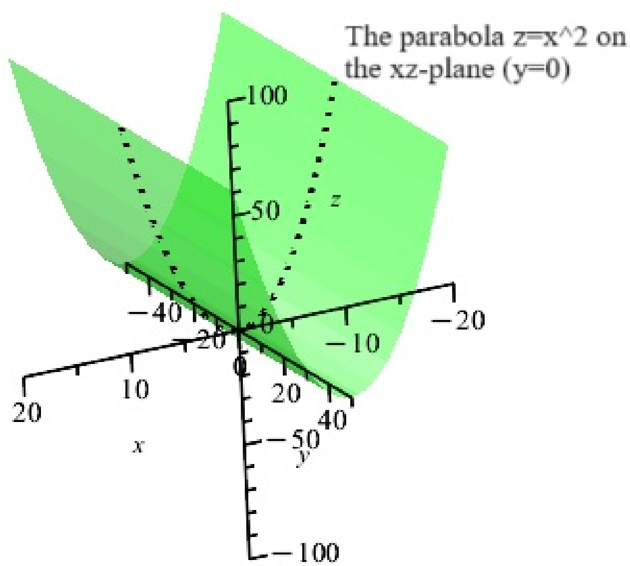
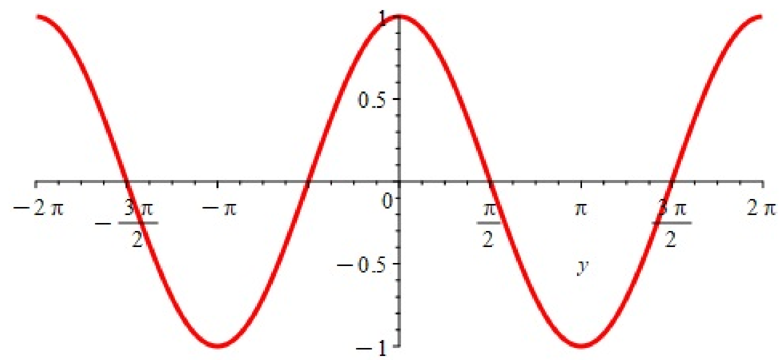
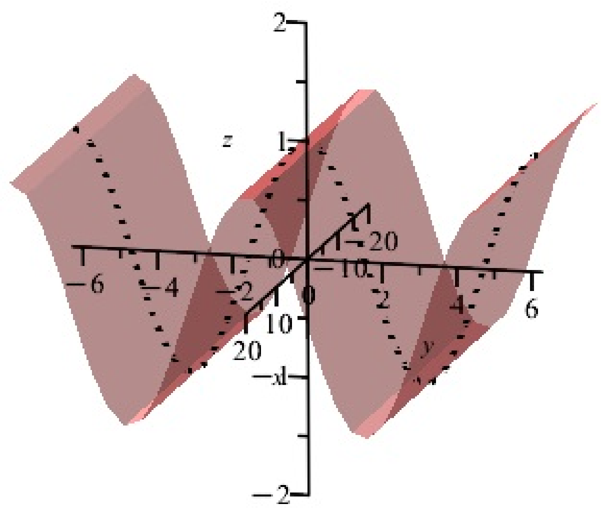


Problem 4. Consider the equation $z = \cos(y)$.
(a) Describe the type of curve this equation represents in $\mathbb{R}^2$ (on the yz-plane). Draw a rough sketch of the curve. **HINT:** Let the z-axis be vertical.
(b) Draw a rough sketch of the surface with axes in standard position in $\mathbb{R}^3$.

(a) The equation represents the curve of cosine $z = \cos(y)$ on the yz-plane.



(b)



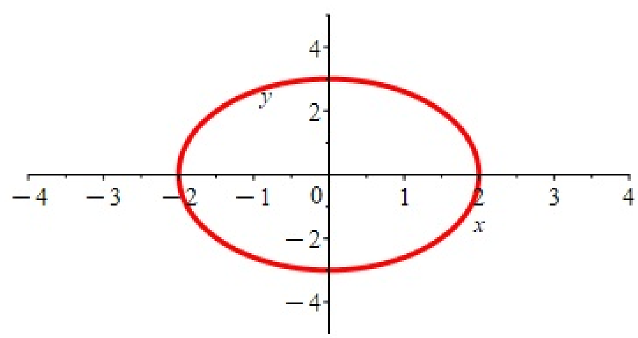
Problem 5. Generally, the equation

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1,$$

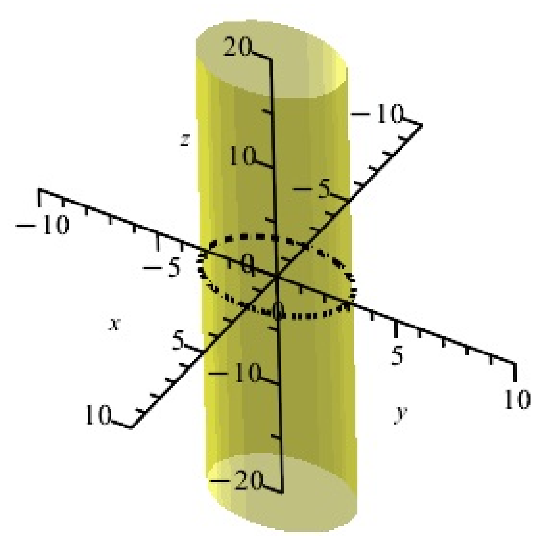
where a, b, h, and k are constants, represents the equation of an **ellipse** centered at (h, k) in $\mathbb{R}^2$. Think of a as the "horizontal radius" and b as the "vertical radius" of the ellipse, respectively.

Consider the equation $\frac{x^2}{4} + \frac{y^2}{9} = 1$.
(a) Draw a rough sketch of the ellipse in $\mathbb{R}^2$ (on the xy-plane).
(b) Draw a rough sketch of the surface with axes in standard position in $\mathbb{R}^3$.
This surface is called an elliptic cylinder in $\mathbb{R}^3$.

(a) The equation can be rewritten as $x^2/2^2 + y^2/3^2 = 1$. It represents the ellipse with horizontal radius 2 and vertical radius 3 on the xy-plane.



(b)



Problem 6.
(a) Find an equation of the sphere with center $(1, 0, -2)$ and radius 3.
(b) What is the intersection of the sphere with the plane $x = 3$? Write its equation and describe the curve.

(a) $(x-1)^2 + y^2 + (z+2)^2 = 9$

(b) We can obtain the equation of the intersection of the sphere with the plane $x = 3$ by substituting $x = 3$ into the equation of the sphere. We have

$$(3-1)^2 + y^2 + (z+2)^2 = 9 \quad \Rightarrow \quad y^2 + (z+2)^2 = 5.$$

The equation $y^2 + (z+2)^2 = 5$ represents the a circle of radius $\sqrt{5}$ centered at the point $(0, -2)$ on the yz-plane.

Problem 7.
(a) Verify all of your answers in part (b) for problems 1-6 by plotting the surfaces/curves in Maple.

See the following pages.

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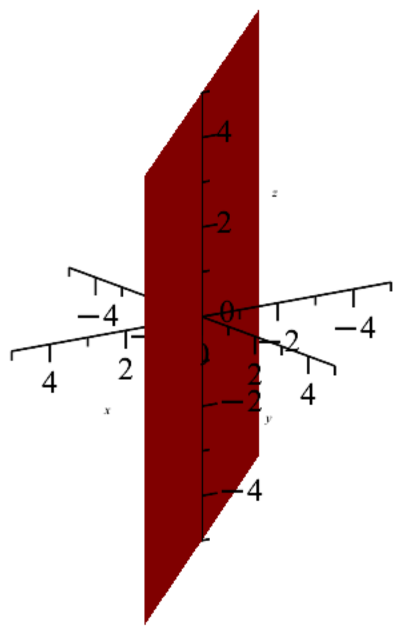
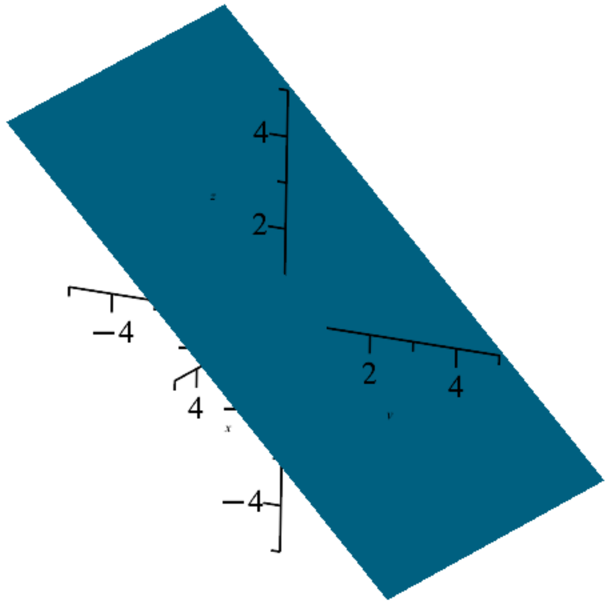
> $with(plots):$

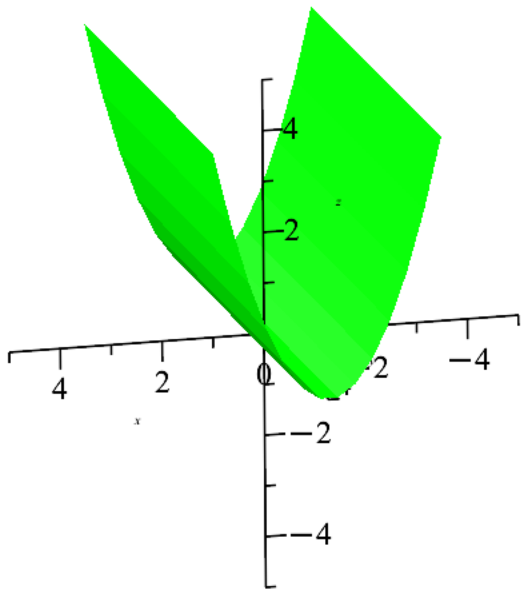
> $implicitplot3d(y=x, x=-5..5, y=-5..5, z=-5..5, axes=normal, style=Surface, color=red)$



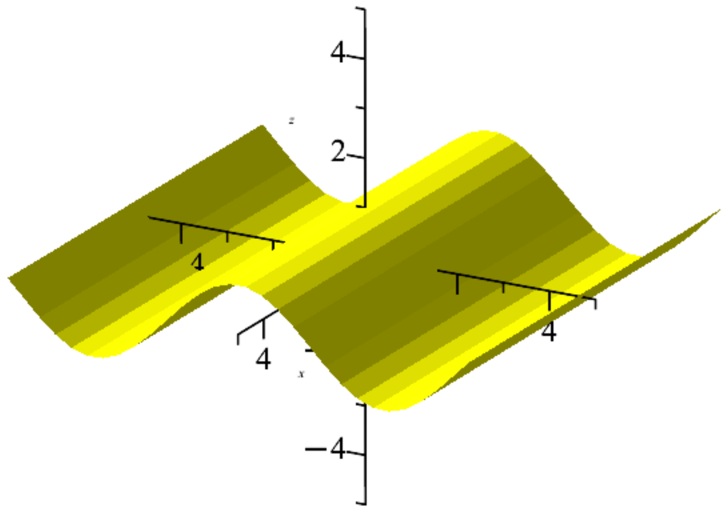
> $implicitplot3d(y+z=1, x=-5..5, y=-5..5, z=-5..5, axes=normal, style=Surface, color=deepskyblue)$



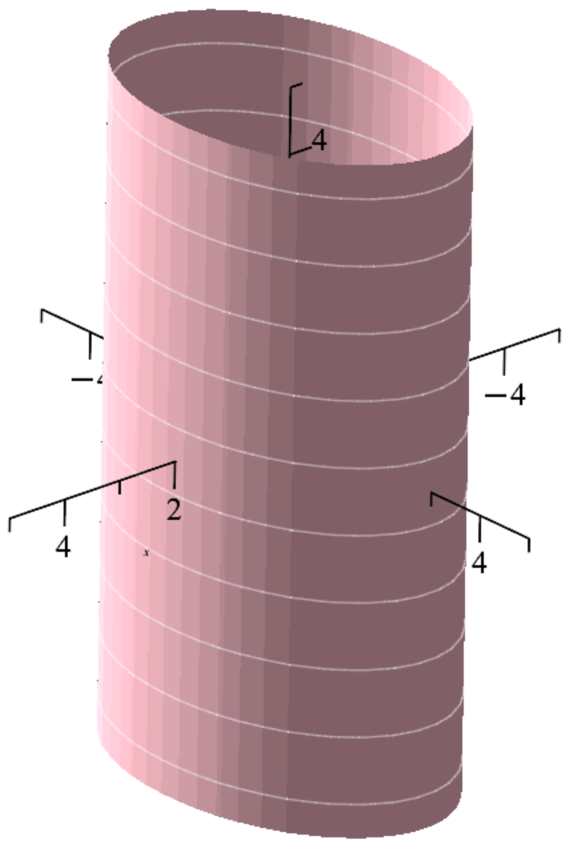
> $implicitplot3d\left(z = x^2, x = -5\,..5, y = -5\,..5, z = -5\,..5, axes = normal, style = Surface, color = green\right)$



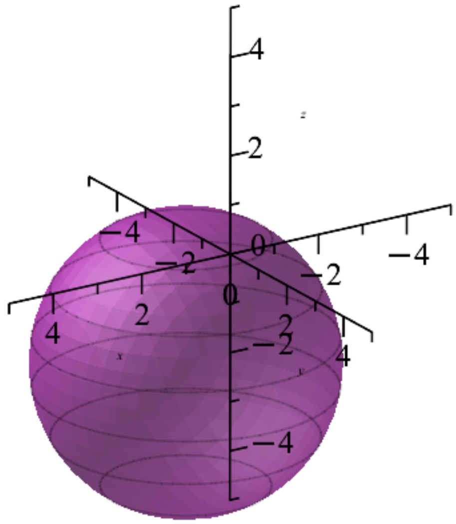
> *implicitplot3d*(*z* = cos(*y*), *x* = −5 ..5, *y* = −5 ..5, *z* = −5 ..5, *axes* = *normal*, *style* = *Surface*, *color* = *yellow*)



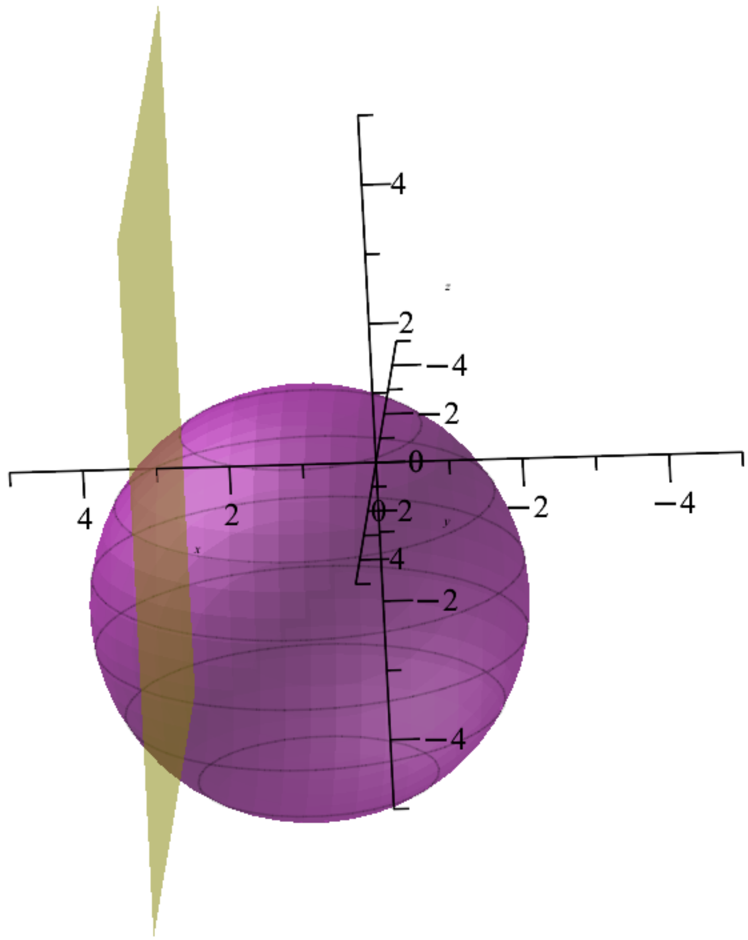
> $implicitplot3d\left(\frac{x^2}{4}+\frac{y^2}{9}=1, x=-5..5, y=-5..5, z=-5..5, axes=normal, style=SurfaceContour, color=pink\right)$



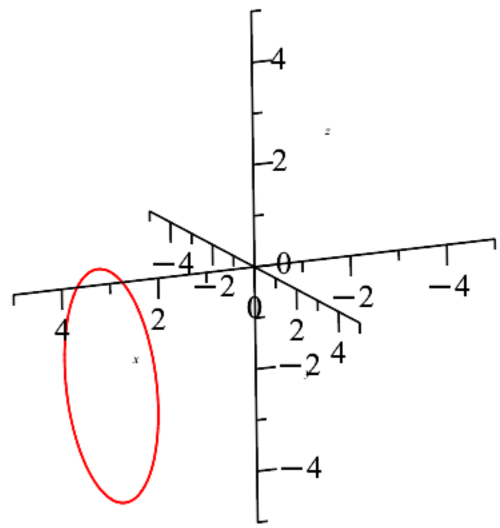
> $implicitplot3d\left((x-1)^2 + y^2 + (z+2)^2 = 9, x = -5..5, y = -5..5, z = -5..5, axes = normal, style = SurfaceContour, color = purple, transparency = 0.5 \right)$



> $implicitplot3d\left(\left[(x-1)^2+y^2+(z+2)^2=9,\ x=3\right],\ x=-5..5,\ y=-5..5,\ z=-5..5,\ axes = normal,\ style=\left[SurfaceContour,\ Surface\right],\ color=\left[purple,\ yellow\right],\ transparency=0.5\right)$



> $intersectplot\left((x-1)^2+y^2+(z+2)^2=9,\ x=3,\ x=-5..5, y=-5..5, z=-5..5, axes=normal,\ color=red\right)$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