Section 12.2: Vectors

Problem 1. Find the vector in $\mathbb{R}^3$ that has the opposite direction as $\langle 6, 2, -3\rangle$ and has length 4.

Section 12.3: The Dot Product

Problem 2. Use vectors to determine whether the triangle with vertices $P = (1, -3, -2)$, $Q = (2, 0, -4)$, and $R = (6, -2, -5)$ is a right triangle.

Problem 3. In class, we learned that vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal (perpendicular) if and only if $\mathbf{a} \bullet \mathbf{b} = 0$.

Vectors $\mathbf{a}$ and $\mathbf{b}$ are parallel if and only if the angle θ between them is 0 or π. That is, $\mathbf{a}$ and $\mathbf{b}$ are parallel if and only if

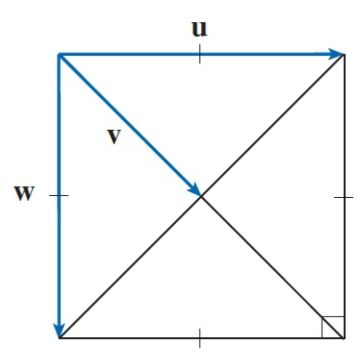
$$\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}| \cdot |\mathbf{b}| \cdot \cos(\theta) = |\mathbf{a}| \cdot |\mathbf{b}|(\pm 1) \quad \Leftrightarrow \quad \frac{\mathbf{a} \bullet \mathbf{b}}{|\mathbf{a}| \cdot |\mathbf{b}|} = \pm 1,$$

since $\cos(0) = 1$ and $\cos(\pi) = -1$.

Let $\mathbf{a} = \langle 1, 2, 3\rangle$, $\mathbf{b} = \langle 0, 1, 3\rangle$, and $\mathbf{c} = \langle 2, -1, -1\rangle$. Determine whether the following pairs vectors are orthogonal, parallel, or neither.
(a) $\mathbf{a}$, $\mathbf{b}$ (b) $\mathbf{a}$, $\mathbf{c}$, (c) $\mathbf{a}$, $\mathbf{b} + \mathbf{c}$ (d) $2\mathbf{a}$, $\mathbf{b}$.

Problem 4. Which of the following expressions are meaningful? Which are meaningless? Explain.
(a) $(\mathbf{a} \bullet \mathbf{b}) \bullet \mathbf{c}$, (b) $|\mathbf{a}|(\mathbf{a} \bullet \mathbf{c})$, (c) $\mathbf{a} \bullet \mathbf{b} + \mathbf{c}$

Problem 5. If $\mathbf{u}$ is a unit vector, find $\mathbf{u} \bullet \mathbf{v}$ and $\mathbf{u} \bullet \mathbf{w}$.
NOTE: The dashes on the sides of the figure below indicate the figure is a square. Assume that the angles between the diagonal lines are each $\pi/2$ (90 degrees).



Problem 6. Find a unit vector that is orthogonal to both of the vectors $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} + \mathbf{k}$.

HINT: Let $\mathbf{u} = \langle u_1, u_2, u_3\rangle$ be the unit vector. Take its dot product with $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} + \mathbf{k}$ and set up a system of two equations to determine the entries of $\mathbf{u}$.