MAT 2500 (Dr. Fuentes) — Worksheet 1 - Section 12.1

Section 12.1: 3-Dimensional Coordinate Systems

Problem 1. Consider the equation $y = x$.
(a) Describe the type of curve this equation represents in $\mathbb{R}^2$ (on the xy-plane). Draw a rough sketch of the curve.
(b) Describe the type of surface this equation represents in $\mathbb{R}^3$. Draw a rough sketch with axes in standard position.

Problem 2. Consider the equation $y + z = 1$.
(a) Describe the type of curve this equation represents in $\mathbb{R}^2$ (on the yz-plane). Draw a rough sketch of the curve. **HINT:** Let the z-axis be vertical.
(b) Describe the type of surface this equation represents in $\mathbb{R}^3$. Draw a rough sketch with axes in standard position.

Problem 3. Consider the equation $z = x^2$.
(a) Describe the type of curve this equation represents in $\mathbb{R}^2$ (on the xz-plane). Draw a rough sketch of the curve. **HINT:** Let the z-axis be vertical.
(b) Draw a rough sketch of the surface with axes in standard position in $\mathbb{R}^3$. **This surface is called a <u>parabolic cylinder</u> in $\mathbb{R}^3$.**

Problem 4. Consider the equation $z = \cos(y)$.
(a) Describe the type of curve this equation represents in $\mathbb{R}^2$ (on the yz-plane). Draw a rough sketch of the curve. **HINT:** Let the z-axis be vertical.
(b) Draw a rough sketch of the surface with axes in standard position in $\mathbb{R}^3$.

Problem 5. Generally, the equation

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1,$$

where a, b, h, and k are constants, represents the equation of an **ellipse** centered at (h,k) in $\mathbb{R}^2$. Think of a as the "horizontal radius" and b as the "vertical radius" of the ellipse, respectively.

Consider the equation $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$.
(a) Draw a rough sketch of the ellipse in $\mathbb{R}^2$ (on the xy-plane).
(b) Draw a rough sketch of the surface with axes in standard position in $\mathbb{R}^3$. **This surface is called an <u>elliptic cylinder</u> in $\mathbb{R}^3$.**

Problem 6.
(a) Find an equation of the sphere with center $(1, 0, -2)$ and radius 3.
(b) What is the intersection of the sphere with the plane $x = 3$? Write its equation and describe the curve.

Problem 7.
(a) Verify all of your answers in part (b) for problems 1-6 by plotting the surfaces/curves in Maple.