MAT 2500 (Dr. Fuentes) Worksheet 7 - Sections 14.2 & 14.3

Section 14.2: Limits & Continuity of 2-Variable Functions

Problem 1. Show that the following limits **do not exist**.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^2\sin^2(x)}{x^4+y^4}$ (b) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{y-x}{1-y+\ln(x)}$ (c) $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz^2+xz^2}{x^2+y^2+z^2}$

Problem 2.

(a) Show that $\displaystyle\lim_{(x,y)\to(1,0)} (x-1)^2\cos\left(\frac{1}{y}\right) = 0.$ (b) Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0.$

Problem 3. Determine the set of points at which the function is continuous.

(a) $F(x,y) = \dfrac{xy}{1+e^{x-y}}$ (b) $g(x,y) = \dfrac{e^x+e^y}{e^{xy}-1}$

(c) $f(x,y) = \begin{cases} \dfrac{xy}{x^2+xy+y^2} & \text{if } (x,y)\neq(0,0) \\ 0 \quad \text{if } (x,y)=(0,0). \end{cases}$

Section 14.3: Partial Derivatives

Problem 4. Find the partial derivatives of the following functions:

(a) $f(x,y) = y(x+x^2y)^5$

(b) $u(r,\theta) = \sin\left(\dfrac{r\cos(\theta)}{\theta^2}\right)$

(c) $w = xy^2e^{-xz}$

Problem 5. Let $f(x,y,z) = x^{yz}$. Find $\dfrac{\partial^2 f}{\partial z\partial x}$ at the point $(e,1,0)$.

HINT: Remember the following differentiation formula from Calculus I: $\dfrac{d}{dt}b^t = \ln(b)b^t$, where $b>0$

Problem 6. Find all the second partial derivatives of $z = \dfrac{y}{2x+3y}$.

In class, we found that $\dfrac{\partial z}{\partial x} = -\dfrac{2y}{(2x+3y)^2}$ and $\dfrac{\partial z}{\partial y} = \dfrac{2x}{(2x+3y)^2}$.

Problem 7. Find the indicated partial derivative. **HINT:** You can change the order of differentiation using Clairaut's Theorem.

$w = \sqrt{u+v^2} + u\sin^2(t)$; $\quad \dfrac{\partial^4 w}{\partial v^2\partial t\partial u}$.

Problem 8. Let $f(x,y) = 16-4x^2-y^2$.
(a) Use trace curves to determine the graph of f.
(b) Find $f_x(1,2)$ and $f_y(1,2)$ and interpret these numbers as slopes of tangent lines to trace curves.