Section 14.7: Maximum and Minimum Values

Second Derivatives Test Suppose the second partial derivatives of f are continuous for all points (x,y) "near" (a,b), and suppose that

$$f_x(a,b) = 0 = f_y(a,b).$$

Let

$$D = D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$$

(a) If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a,b)$ is a local minimum
(b) If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a,b)$ is a local maximum
(c) If $D < 0$, then (a,b) is a saddle point of f
(d) If D = 0, then the test is inconclusive for the point (a,b).

Problem .1 Find the local extreme value(s) and the saddle point(s) of the function, if they exist.
(a) $F(x,y) = xy - x^2y - xy^2$ (b) $f(x,y) = e^x\cos(y)$

(c) $g(x,y) = y^2 - 2y\cos(x), \quad -1 \le x \le 7$

Problem 2. The base of an aquarium (open top) with volume 800 in^3 is made of stone and the sides are made of glass. If the stone costs five times as much (per in^2) as the glass, find the dimensions of the aquarium that minimize the cost of the materials.

HOW TO FIND ABSOLUTE MAX AND MIN VALUES OVER A CLOSED AND BOUNDED SET

9 To find the absolute maximum and minimum values of a continuous function f on a closed, bounded set D:

1. Find the values of f at the critical points of f in D.
2. Find the extreme values of f on the boundary of D.
3. The largest of the values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Problem 3. Find the absolute maximum and minimum values of the function on the given set D.

(a) $f(x,y) = x^2 + y^2 + x^2y + 4, \qquad D = \{(x,y) \mid |x| \le 1, |y| \le 1\}.$

(b) $g(x,y) = xy^2, \qquad D = \{(x,y) \mid x \ge 0, y \ge 0, x^2 + y^2 \le 3\}.$

Section 15.1: Double Integrals over Rectangles

Problem 4. Calculate the following double integrals.

(a) $\displaystyle\iint_R \frac{\ln(y)}{xy}\,dA$, where $R = [1,3] \times [1,5]$.

(b) $\displaystyle\int_0^1 \int_0^1 \frac{x}{1+xy}\,dy\,dx$

HINT: Use a u-substitution then later you will need to apply integration by parts.

(c) $\displaystyle\iint_R xy\sqrt{x^2+y^2}\,dA$, where $R = [0,1] \times [0,1]$. **HINT:** Use the u-substitution $u = x^2 + y^2$.

(d) $\displaystyle\int_{-2}^{-1} \int_0^2 \int_0^1 \frac{x^2 e^y}{z}\,dx\,dy\,dz$

Problem 5. Find the volume of the solid in the first octant bounded by the parabolic cylinder $z = 16 - x^2$ and the plane $y = 5$.